

On Some Physical Properties of a Unified Lepton-Hadron Field Model with Composite Bosons

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In preceding papers a lepton-hadron unified field model was introduced by means of a third order nonlinear spinorfield equation. In this paper an improved interpretation for this model is given which tries to incorporate the advantages of various current matter models and to avoid their drawbacks. In particular charge and lepton number are introduced, while the extended unstable baryon states are distinguished from lepton states by an intrinsic parity. A theorem is derived which allows a biunique decomposition of the nonlinear higher order spinorfield equation into nonlinear first order spinorfield equations and the simultaneous introduction of a permutation group of the subfields. These subfields are identified as pseudo-color fields.

Unified field models are defined by quantum field theories in which all particles are assumed to be bound states of elementary fermion fields; in particular gauge bosons are assumed to be fused from two or more fermion fields, in contrast to gauge field theories where these gauge bosons are considered to be elementary.

The first step towards unified field models was done by de Broglie [1] who assumed the photon to be fused from two neutrinos. Later on, Heisenberg [2] proposed a nonlinear spinorfield as a unified universal field model. At that time lepton and baryon conservation were considered as indispensable. But it was not possible to deduce a separate lepton and baryon conservation law from this spinorfield equation.

The rise of the grand unification gauge theories which were initiated by Pati and Salam [3] has changed the situation drastically. In these theories it is assumed that the only exact symmetries are gauge symmetries, such as SU_3 color and U_1 , which guarantee color- and electromagnetic charge conservation resp., whereas either baryon conservation or both baryon and lepton conservation are violated, cf. Ellis [4]. In the meantime great experimental efforts have been made in order to measure such decay processes.

On the other hand, even the SU_3 color invariance is partly assumed not to be an elementary symmetry as for instance in the subquark models of Harari [5]

and Grosser [6] or in the unconventional electromagnetic matter model of Barut [7]. This situation offers the possibility of a further development of unified field models.

In this paper we discuss a unified field model of matter by means of an improved interpretation of a lepton-quark model based on a higher order nonlinear spinorfield equation which was proposed by Stumpf [8]. For this type of field equations a practically complete relativistic dynamics for composite particles were developed by Stumpf and coworkers [9] which is called functional quantum theory. Hence in the following we need not discuss the formulation of a relativistic composite particle dynamics connected with such unified field models. Rather, we discuss some basic properties of the field equation and the state representation which explain the physics contained in this model.

We assume the basic field to be a spinor-isospinor field $\psi_{\alpha k}(x)$ where $k=1, 2$ means the isospin index and $\alpha=1, 2, 3, 4$ the spinor index. For simplicity we consider a parity symmetric model which implies the supposition that parity violation is not the clue for understanding the fundamental properties of matter formation and reactions. We propose the following higher order nonlinear spinorfield equation

$$\begin{aligned} & [(-i\gamma^\mu \partial_\mu + \mu_k^1)(-i\gamma^\nu \partial_\nu + \mu_k^2) \\ & \cdot (-i\gamma^\rho \partial_\rho + \mu_k^3)]_{\alpha\beta} \psi_{\beta k}(x) \\ & = g \gamma^\mu_{\alpha\beta} \psi_{\beta r}(x) \bar{\psi}_{rs}(x) \gamma_\mu \gamma_\delta \psi_{\delta t}(x) \\ & \cdot [\tfrac{3}{4} \delta_{kr} \delta_{st} + \tfrac{1}{4} \tau_{kr} \cdot \tau_{st}]. \end{aligned} \quad (1)$$

The isospin vertex part was taken over from Dürr [10]. For $\mu_k^2 = \mu_k^3$ we obtain the original model [8] where dipole ghosts were interpreted as quarks.

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For an improved interpretation of (1) we try to combine some remarkable features and advantages of three different types of matter models and to avoid their drawbacks. The three types of models are

- i) the magnetic (electromagnetic) model of Barut and Barut and coworkers,
- ii) the quark model of Gell-Mann and Zweig and its extension to quantum chromodynamics,
- iii) the first order nonlinear spinorfield model of Heisenberg, Dürr and coworkers and the lepton model of Saller.

We first discuss these models in turn.

Barut and Barut and coworkers [7] proposed a model which is based on three stable fermions p , e , ν and the photon γ . The hierarchy of elementary particles and forces is explained by the formation of magnetically bound states and resonances resp., and by the various interactions arising from the extended structures of the particles. This model is a straight-forward extension of the quantum-mechanical low energy atomic and molecular model of matter to high energy phenomena. The drawbacks of Barut's model are that the proton has definitely no structure and that a manageable relativistic dynamics of composite particles based on renormalized quantum electrodynamics is not available.

The proton structure is correctly described in the quark model of Gell-Mann and Zweig [11]. Its formulation by means of a quantized gauge theory leads to quantum chromodynamics and to grand unification gauge theories resp. The drawbacks of the gauge theory approach are that so far neither confinement has been proven nor a relativistic dynamics of composite particles has been derived from the path integral quantization.

Heisenberg [2] proposed a first order nonlinear spinorfield equation with Pauli-Villars dipole ghost regularization, where the basic particles are p , n , while the leptons are vaguely identified with dipole ghosts. Saller [12] gave the reverse interpretation of this equation as a lepton model where the basic particles are e , ν , while quarks are vaguely identified with dipole ghosts. The drawbacks of this approach are that no separate dipole ghost dynamics is derived, in the Heisenberg version the proton is point-like and that the various generations remain unexplained. In addition, this approach suffers from

indefinite metric and no manageable and selfconsistent dynamics for relativistic composite particles has been derived.

Concerning the problems of relativistic composite particle dynamics which simultaneously occur in i), ii) and iii) we refer to functional quantum theory [9] which offers a solution of these problems for the model under consideration. Hence we have to discuss only the general physical ideas of i), ii), and iii) which seem to be of interest for the interpretation of (1). In order to do this we collect the advantages of the approaches i)–iii) which should in our opinion be shared by (1). These advantages are

- i) the explanation of the higher generations as bound states or resonance states resp. of the particles of the first generation and partly the assumption that particles are composed by those constituents into which they decay;
- ii) the extended proton structure and proton decay, the color symmetry, the color singlet rule and confinement, the charge and lepton number conservation;
- iii) the composite boson structure and the interpretation of the isospinor as an electroweak isospinor with basic particles e and ν which are point-like.

If we try to incorporate these features into the physical interpretation of (1) we obtain the following (incomplete) results:

Definition of Lepton and Baryon States

The vertex operator of (1) contains the unit operator in isospin space and the scalar product of two isovectors. In the corresponding Lagrangian both expressions are forminvariant with respect to the full isospin rotation group $SU(2)$. The kinetic term of (1), however, breaks the isospin invariance due to the masses μ_k^s for different isospin degrees of freedom. In the Lagrangian this term shows only an invariance with respect to rotations around the third axis, i.e. rotations of the kind $\exp(\frac{1}{2}i\epsilon\tau_3)$. Due to the forminvariance of the Lagrangian with respect to these transformations we can always choose basis vectors which are eigenvectors of the generator τ_3 . Hence it is possible to define the eigenvalues of the charge operator $Q = e(T_3 + 1)/2$ as a good quantum

number, where e is the negative charge of the electron and where T_3 is the general generator of isospin rotations around the third axis. For one-fermion states which start with $\langle 0 | \psi(x) | f \rangle$ we get the charge states e and 0 .

In addition, (1) admits the general phase transformation $\psi'(x) = \exp(i\theta L) \psi(x)$ which leaves the corresponding Lagrangian form-invariant and gives rise to a general fermion conservation law. According to i), ii), and iii) we primarily try to interpret (1) as a lepton model. Hence we identify the eigenvalues of L with the lepton number and choose for the states, starting with $\langle 0 | \psi(x) | f \rangle$ the value $L = 1$. According to ii) the proton must have a structure. Hence we cannot identify any fermion state of the kind $\langle 0 | \psi(x) | f \rangle$ with the proton. Rather we had to start with $\langle 0 | \bar{\psi}(x) \psi(y) \bar{\psi}(z) | p \rangle$ which has the lepton number $L = -1$. If we define the baryon number B by $B := -L$, then for any reaction of baryons with leptons we have $L - B' = L + L' = \text{constant}$, i.e. in this definition the difference between lepton number and baryon number is a conserved quantity. This reduced conservation law also occurs in grand unification SU(5) theories and in the Harari model. It means that the proton and other baryons can decay into leptons. Now what is the difference between a proton and a positron? This difference follows from construction. The corresponding functional equation allows for leptons starting with $\langle 0 | \psi(x) | l \rangle$ completely point-like solutions, whereas the protons are genuine nonlocal states. As, apart from the different masses, formally a proton and a positron have the same quantum numbers, it cannot, however, be excluded a priori that $\langle 0 | \psi(x) | p \rangle \neq 0$, which contradicts our assumption. The solution of this problem comes directly from the dynamical equations of the quantized version of the theory in functional space. Here we do not go into details of this treatment but give only the result. For the same quantum numbers, apart from masses, the dynamical equations possess two different types of solutions which are distinguished by

$$\langle 0 | \psi(x) \bar{\psi}(x) \psi(x) | l \rangle \neq 0 \quad \text{and}$$

$$\langle 0 | \psi(x) \bar{\psi}(x) \psi(x) | p \rangle = 0,$$

resp. In the latter case the dynamical equations admit a solution with $\langle 0 | \psi(x) | p \rangle \equiv 0$, whereas in the former case $\langle 0 | \psi(x) | l \rangle \neq 0$ is a stringent con-

dition for getting the solution. Hence the protons are dynamically characterized by a subsidiary condition on the wave function at the origin which must not be shared by leptons. Such a subsidiary condition can be expressed by the intrinsic parity of an extended object, i.e. a definition which is meaningless for point particles. The intrinsic parity therefore allows a discrimination between extended and point particles and explains the difference between leptons and baryons.

Derivation of Pseudo-Color and Pseudo-Color Singlet Rule

To any charge state of (1) three different masses μ_k^i , $1 \leq i \leq 3$ are attached. These masses are connected with corresponding subfields. For their discussion the charge index k does not play any role and we omit it in the following. We use the definitions

$$D_i := (-i\gamma^\mu \partial_\mu + \mu^i);$$

$$G_i := D_i^{-1}; \quad 1 \leq i \leq 3;$$

$$F := G_1 G_2 G_3.$$

Then the following theorem holds:

Theorem. Suppose that the relation

$$\lambda_1 G_1 + \lambda_2 G_2 + \lambda_3 G_3 = gF \quad (4)$$

holds with λ_i , $i = 1, 2, 3$ real numbers.

- i) Let $\psi \equiv \psi(x)$ be a bound state solution (\equiv solution with homogeneous boundary conditions) of the equation

$$D_1 D_2 D_3 \psi = gV[\psi], \quad (5)$$

where $V[\psi]$ is a nonlinear interaction term, and let subfields $\varphi_i \equiv \varphi_i(x)$, $i = 1, 2, 3$ be defined by

$$\varphi_i := g^{-1} \lambda_i D_h D_j \psi, \quad (6a)$$

$$i, h, j = 1, 2, 3 \text{ cycl. perm.},$$

then the relation

$$\psi = \varphi_1 + \varphi_2 + \varphi_3 \quad (7a)$$

is satisfied and the subfields are bound state solutions of the equations

$$D_i \varphi_i = \lambda_i V \left[\sum_{i=1}^3 \varphi_i \right], \quad i = 1, 2, 3. \quad (8)$$

- ii) Let $\varphi_i \equiv \varphi_i(x)$, $i = 1, 2, 3$ be bound state solutions of the equations

$$D_i \varphi_i = \lambda_i V \left[\sum_{i=1}^3 \varphi_i \right], \quad i = 1, 2, 3 \quad (8)$$

and define $\psi \equiv \psi(x)$ by

$$\psi := \varphi_1 + \varphi_2 + \varphi_3, \quad (7b)$$

then the relations

$$\begin{aligned} \varphi_i &= g^{-1} \lambda_i D_h D_j \psi, \\ i, h, j &= 1, 2, 3 \text{ cycl. perm.} \end{aligned} \quad (6b)$$

are satisfied and ψ is a bound state solution of the equation

$$D_1 D_2 D_3 = g V[\psi]. \quad (5)$$

- iii) A biunique map between bound state solutions of the equations (5) and (8) is established by (6a) or (7b) resp.

Proof. i) From (4) we get the relation

$$\lambda_1 D_2 D_3 + \lambda_2 D_1 D_3 + \lambda_3 D_1 D_2 = g \quad (9)$$

and using (6a) and (9) we obtain by straightforward calculation (7a). Furthermore, by means of (6a) and (5) the application of D_i to φ_i , $i = 1, 2, 3$ gives the set of Equations (8).

- ii) From (8) we get for bound state solutions

$$\varphi_i = \lambda_i G_i V \left[\sum_{i=1}^3 \varphi_i \right], \quad i = 1, 2, 3. \quad (10)$$

By summation over i and using (4) and (7b) we obtain the equation

$$\psi = g F V[\psi] \quad (11)$$

and for homogeneous boundary conditions the Equation (5). Furthermore, from (8) it follows

$$\lambda_i^{-1} D_i \varphi_i = \lambda_j^{-1} D_j \varphi_j. \quad (12)$$

If we now apply $\lambda_2^{-1} \lambda_3^{-1} D_2 D_3$ to (7b) by means of (12) the relation

$$\begin{aligned} \lambda_2^{-1} \lambda_3^{-1} D_2 D_3 \psi \\ = [\lambda_2^{-1} \lambda_3^{-1} D_2 D_3 + \lambda_1^{-1} \lambda_3^{-1} D_1 D_3 \\ + \lambda_2^{-1} \lambda_1^{-1} D_2 D_1] \varphi_1 \end{aligned} \quad (13)$$

can be derived. Multiplication of (13) by $\lambda_1 \lambda_2 \lambda_3$ and use of (9) gives (6b) for $i = 1$, etc.

- iii) Let $C := \{\psi^\alpha, \alpha = 1, 2, \dots\}$ be the set of bound state solutions of (5) and let

$$K := \{\varphi_i^\beta, i = 1, 2, 3, \beta = 1, 2, \dots\}$$

be the set of bound state solutions of (8). Then according to the proof i) by (6a) C can be mapped onto a set of bound state solutions K' of (8). As K contains all bound state solutions of (8) by definition, we have $K' \subset K$. On the other hand, if according to the proof ii) the set K is mapped onto a set C' of bound state solutions of (5), we have $C' \subset C$. Now from $K \Rightarrow C'$ it follows $C' \Rightarrow K^* \equiv K$ and therefore $C \Rightarrow K'$ gives $K \subset K'$, as C contains more elements than C' and therefore the map K' must also contain more elements than the K^* of C' . Hence we have $K' \subset K \subset K'$ and from this it follows $K' \equiv K$. Q.E.D.

It should be noticed that (4) is not an additional assumption. Rather this relation is equivalent to the so-called Pauli-Villars regularization and can be proved directly for the operators D_i defined by (2), cf. for instance [14].

Furthermore, the use of the biunique map for the fields ψ and φ_i only for bound states does not lead to an essential restriction with respect to the quantized version of the theory in functional space, because in a theory of this kind the undressed particles which correspond to the free field operators of the subfields φ_i are not allowed to occur as ingoing or outgoing asymptotic scattering states. The formalism is constructed in such a way that only dressed particles can occur in the asymptotic scattering states.

The equivalence of the solutions and corresponding Eqs. (5) and (8) was first discussed for free fields, i.e., $V[\] \equiv 0$. For this case Wildermuth [13] and Pais and Uhlenbeck [14] derived the corresponding equivalence. The first step to show the equivalence of (5) and (8) for the nonlinear case, i.e., $V[\] \neq 0$ was done by Stumpf [8]. But in [8] only a unique map is discussed, while the more important step of a biunique map is given in the theorem above.

If the decomposition (7a) is substituted into (1) it is obvious that (1) is invariant against any permutation of the three subfields, since the linear superposition (7a) of these subfields and the operator $D_1 D_2 D_3$ are invariant against these permutations. A state being invariant under the action of a complete set of group operations must be a singlet state, i.e., belongs to a one-dimensional representation of this group. Since any quantum state of (1) is represented by a linear combination of direct products of ψ and $\bar{\psi}$ applied to the invariant vacuum

state, (1) together with (7a) selects one-dimensional state representations of the permutation group S_3 with respect to the subfields $\varphi_1, \varphi_2, \varphi_3$. In this way the physical content of (1) is revealed as being equivalent to a highly symmetric subfield dynamics. The proton is then constructed in its lowest order representation, i.e., without polarization cloud, as a direct product of three singlet representations of the subfields. Although in this model the proton consists of lepton fields, its general structure is similar to that of quark theories, where the subfields are interpreted as colored quark fields. It is therefore reasonable to consider the decomposition (7a) as the introduction of a "pseudo" color in the model (1) which produces similar effects as the color in quark theories, but additionally gives an explanation of "pseudo" color as an effect of regularization, since (1) is a selfregularizing system. Finally, it should be noticed that the invariance of (1) against permutations with respect to the decomposition (7a) is reflected in the system (8) as an invariance against permutations with respect to the indices i and masses μ_k^i .

Confinement and Unitarization

The assumption (4) is equivalent to a Pauli-Villars regularization condition. This implies that (1) or (5) resp., and (8) contain ghost particles for $g=0$ or $V[\] \equiv 0$ resp. If (1) or (5) and (8) are studied within the framework of a coupling theory, these ghost particles are not allowed to occur as ingoing or outgoing free particles. The suppression of free ghost particles is usually achieved by unitarization. If ghost or multipole ghost particles are identified with quarks, then unitarization can be equivalently called confinement. The term quarks for ghost states has not to be taken literally, as quarks and ghost states have in common only the confinement property. It has, however, to be observed that unitarization or confinement are connected in this way with the interaction representation. The functional quantum theory works beyond this representation. In particular by the "pseudo" color singlet rule

which is automatically incorporated in the state representation only a symmetric mixture (7a) of ghost fields and physical fields occurs. Even this mixture is of no physical relevance as any physical particle is described by a bound state solution of the corresponding functional equation. Hence the final decision about the physical spectrum can only be made with respect to the solutions of the corresponding functional equation and their norm expressions.

The ratio of $\sigma(e^+e^- \rightarrow \text{hadrons})$ and $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ probably indicates that quarks should have fractional charges. In gauge theories of leptons and quarks it is then assumed that leptons have integer charges while quarks have fractional charges. In contrast to gauge theories such an assumption is not possible in unified field models. Either the fundamental fermion fields have all integer charges or they have all fractional charges. The case of integer charges was discussed above. The case of fractional charges is treated in the model of Grosser and Lauxmann [6]. The $T(x)$ fermion field of this model corresponds to the electron field $\psi_1(x)$ of our model, while the $V(x)$ fermion field corresponds to our neutrino field $\psi_2(x)$. In contrast to the $\psi_1(x)$ field, the $T(x)$ field carries a fractional charge $e/3$ while both the $V(x)$ field and the $\psi_2(x)$ field carry no charge. Furthermore, the charge conservation is common to both models while the Rishon number conservation in the fractional charge model corresponds to the lepton number conservation in our model. Accordingly, the mathematical structure of both models is completely equivalent. The differences arise only from the different interpretation of quantum numbers. Therefore, all conclusions which are not touched by the definition of quantum numbers can be taken over from our model to the fractional charge model, in particular, the definition of intrinsic parity and the theorem given above.

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